DEVELOPMENT OF A CAPP SYSTEM FOR HOT EXTRUSION PROCESS

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by

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CERTIFICATE

It is certified that the work contained in the thesis entitled **DEVELOPMENT OF A CAPP SYSTEM FOR HOT EXTRUSION PROCESS** by **Shivendra Shukla** (Roll No. 9510538), has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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Dedicated to -

My Parents

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Synopsis

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The present work deal with development of a Computer aided process planning for HOT EXTRUSION PROCESS. Feature recognition process forms its first part and is developed for some particular designs only. In its second part, an upper bound technique based on the kinematically admissible velocity field is used to determine the forming stress for different regular polygonal sections(the number of sides is from three to infinite) and a rectangular section for given material properties and friction conditions. Using this method the optimal die geometry (and extrusion pressure) which requires the minimal forming stress can be determined.

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Nomenclature

a	half side length of a regular polygonal section as well as rectangular section
b	half side width of a rectangular section
b_i, c_i	constants of polynomial equations determined by the velocity boundary conditions
c	constant determined by the shape of a regular polygonal section
c_1, c_2	constants determined by the shape of a rectangular section
$f_1(z), f_2(Z)$	functions of variable z
f(z)	function of variable z used for the description of the die profile and the streamlines of particles
N	number of sides of a regular polygonal section
n, ϕ, z	n, ϕ, z coordinate system where the domain of the variation of the variables are 0¡=n¡=R_0 , 0¡=$\phi <= \phi_m$ and 0¡=z¡=L . In this case, n is the distance between the extrusion axis and the arbitrary point at die entry

P_av	average pressure on ram
R_0	radius of cylinderical billet
R_L	relative die length ($= L/R_0$)
V_0	velocity of the billet
V_x, V_y, V_z	velocity components in the cartesian coordinate system
W_i	power due to plastic deformation
W_s	power due to die surface friction
x,y,z	cartesian coordinate system
ϵ_{ij}	components of strain rate tensor
σ_0	yield stress of rigid perfectly plastic material
$g(\phi, z)$	functions of variables ϕ and z , derived from the determinant of the jacobian of the coordinate transformation equation
$detJ$	determinant of the jacobian
J	total power consumption inside the die
L	length of die , which is the distance between the entry and the exit shear planes of die

Chapter 1

INTRODUCTION AND LITERATURE SURVEY

1.1 INTRODUCTION

Computer aided design (CAD) engineering and computer aided manufacturing (CAM) engineering have been key components of successful engineering practices since '90s. The integration of CAD and CAM provides a vital link in the form of computer integrated manufacturing system (CIMS). In this regard, feature recognition as an element of computer aided process planning (CAPP), constitutes the major link to bridge CAD and CAM. In this work, some feature recognition methods have been developed for product producible from extrusion process.

Extrusion is one of the most used manufacturing processes in the industry as it is flexible enough to accomodate a wide range of sizes, materials and reduction compared to other processes. Hot extrusion is the process of forcing a heated billet to flow through a die opening of desired shape and is generally used to produce material products of constant cross sections. Cold extrusion on the other hand, reduces the billet to the desired shape at temperature below the crystallization temperature and is used for obtaining higher precision. There are various methods for analysing metal forming problems such as upper bound

technique, finite element technique, slip line method and slab method. Slip line method is used to analyse plane strain deformation in a rigid perfectly plasic isotropic solid, and is therefore more useful for such industrial process as sheet extrusion where deformation can be assumed two dimensional. Slab method can be used for only simple problems, it ignores redundant work and does not give description of strain or stress fields. Finite element technique is a better method than upper bound technique as it is more accurate but it consumes considerable computer time and is more suitable for complex problems.

Upper bound analysis is the most practical technique for simulating metal flow in simple deformation processes. In metal working, it is of great interest since it ensures that a particular operation can actually be carried out without exceeding the predicted load. The critical factor here is that plastic volume should not change, that is the material is incompressible. The results obtained from the use of bounding techniques are usually approximate but they provide a good quantitative feel to the problem.

1.2 PREVIOUS WORK

Feature technology is still at its infancy and the majority of current applications are concentrated on form features. In recent years, form features have been used increasingly in CAD/CAM to define product information. In respect of CAD/CAM integration, one of the main approaches in using form features is feature recognition.

Several different approaches and techniques have been tried in the various feature recognition systems. For example, Woo[9] used the geometry decomposition approach where features are viewed as volumes to be removed by a machining operations. He introduced the concept of cavity feature, that is, features as primitives are the materials to be removed or machined. Dealing with the constructive solid geometry (CSG), modellers Lee and Fu[10] proposed an approach based on principal axis and tree construction to extract and unify manufacturing features. Another approach, proposed by Joshi and

Chang [11] is based on graph theory. They suggested an attributed adjacency graph (AAG) approach which regards a boundary representation model as a graph of attributed paths that defins the adjacency amongst faces and edges.

N.V. Reddy, P.M. Dixit and G.K. Lal [1] proposed combined upper bound and slab method to compare eight different die shapes, namely, streamlines (third and fourth order polynomial, cosine), elliptical, hyperbolic, conical and Blazynski's CRHS . Based on the consideration of total extrusion power (under optimal conditions), they concluded that third and fourth-order polynomial dies and the cosine dies are the best amongst the profiles considered.

It has been shown by Devenpeck and Richmond [2] that the sigmoidal die can provide better mechanical properties of product such as in fatigue life and ductility under well lubricated conditions when compared with other straight, concave, and convex shaped dies. However, they found that theoretically ideal die has a relatively large surface area resulting in an increase of the frictional work done.

Hill [3] pointed out that many different streamlined profiles could be designed for a given reduction of area and in consequence the die profile cannot be uniquely determined. For these reasons Richmond and Morrison[4] proposed the drawing die of minimum length, assuming again a friction free die surface. Devenpeck [5] has tested the efficiency of this die and concluded that a significant characteristic of die profile which minimizes structural damage and work is a zero entrance angle. Zero entrance angle implies that there is no shearing of the material at the inlet of the die. He also concluded that reducing the angle of the exit also appears desirable and thus consideration should be given to the ideal die which has zero exit angle as well as zero entry angle.

Chen and Ling [6] and Avitzur [7] have developed upper bound solutions to extrusion through straight and curved dies. But their analysis is confined to axisymmetric extrusion problems only.

Gunasekera and Hoshino [8] developed a new upper bound solution for the streamlined dies, which is also applicable to other curved dies such as parabolic,

curves and concerned shapes. They concluded that the streamlined die is superior to the straightly converging dies from the aspect of reducing the extrusion load.

1.3 PRESENT WORK

From the research work done till now it can be seen that streamline and cosine dies have been found to be best. At the same time it has been found that optimum die profile may not be uniquely determined, that is, there may be several die profiles which may be optimum. These two observations lead to consideration of only streamlined die profile. At the same time, to keep the die profile flexible subject to its satisfying the boundary conditions. Upper bound analysis done till now have been restricted to axi- symmetric extrusion problem only. This problem has been extended to upper bound analysis of non axi-symmetric problems as in the case of rectangular sections.

In this work, an effort has been made to develop a computer aided process planning (CAPP) system for extrusion process by developing an integrated software package. This package starts with recognizing the features of the design supplied to it. Only for rectangular and regular polygonal section it proceeds further. Based on the upper bound technique, the total power needed for the extrusion process is calculated and is optimized by checking its variation over different die lengths. Die length which corresponds to minimum total power is the optimal die length and hence optimal die geometry is determined.

1.4 ORGANISATION OF THE THESIS

The organisation of the thesis is as follows:

- Chapter 2 discusses the algorithms adopted for the identification of the various shapes.

- Chapter 3 discusses the derivation of the kinematically admissible velocity flow field and the upper bound solutions.
- Chapter 4 discusses the implementation of the proposed CAPP system. about the software package and illustrates it with a few examples.
- Chapter 5 discusses the results obtained and the scope for future work.

Chapter 2

FEATURE RECOGNITION PROCESS

The interfacing of CAD to CAM is a vital step in automated manufacturing. An essential operation in this regard is the recognition of features from the part design and it forms the first stage of a computer aided planning process.

Feature recognition involves receiving geometrical data about the design from data exchange file and processing it. The data exchange file can easily be created from the CAD system, for example Auto CAD, by a set of simple commands. This data exchange file from Auto CAD needs to be slightly modified, by removing some fixed number of initial lines which are not required for the purpose of feature recognition.

A software programme is developed to recognise some particular designs which are among the most frequently used designs in extrusion processes. These are tubular, circular, polygonal, rectangular and I-sections.

Here the methodology involved is to set the geometrical conditions for each shape that must be satisfied by only that shape and by no other. Geometrical constraints for each of above mentioned shapes are give below.

The data exchange file of any shape made with the help of AutoCAD system

contains the (global) coordinates of all the lines (with their starting and end points), circles (with their centre point, radius) etc. While developing the software an assumption is made that the shape has been made in continuity, that is, for a shape consisting of many curves, a new curve always starts from the point where the last curve ends. And it is not that two unconnected curves are first made and then are connected by another curve afterwards. Even if opposite is the case for a shape, some software programmes are available which arrange the data exchange file in a way by which the aforesaid assumption regarding continuity is achieved. In next sections, algorthims for identification of various shapes are discussed.

2.1 CIRCULAR/TUBULAR SECTIONS

In this case, the DFX file has only circles as its constituent elements. If it has only one circle, then the shape is identified to have circular cross section. If the file contains two circles and the centers of the two coincide and radius of one is different than that of the other, then the shape is identified as having tubular cross section.

2.2 RECTANGULAR SECTION

The DFX file has four lines as its constituence elements. Each line starts from the end point of another line. Thus there are four intersecting points. The lengths and included angles between pairs of consecutive lines are calculated. If each pair of intersecting lines make right angle with each other and non intersecting lines are of equal length, then it fulfills the criterion of being a rectangular section.

2.3 POLYGONAL SECTIONS

It involves checking that whether the curve is closed and consists of straight lines which start at the end of previous line.

2.4 I-SECTION

Since the drawing of an I-section can be made in a CAD system starting from any corner and according to it, coordinates of different corner points will follow in that order in the corresponding data exchange file. Thus, the methodology is needed to be a flexible one which irrespective of the way a shape is created, gives the correct result.

The algorithm of identifying the I-section consists of reading of global coordinates of the starting point of the shape and then shifting the origin at that point. In this transformed coordinate system, angle between the lines. starting point of which has already been read and the next one is calculated and noted down. Now, the origin is shifted to the starting point of next line and the angle between this line and following line is computed. This process continues till all the included angles have been covered. If these included angles make analytically predetermined sequence then the design is of an I-section. The procedure is demonstrated below with reference to Fig. 2.1.

Origin is shifted from the global origin 0 to point 1 and x-axis is alligned along line 1-2. Now angle between line 1-2 and line 2-3 is measured with reference to this new cordinate system. Here, for example, it is -90 degrees. The same procedure is adopted for all other lines. The angles that are measured make a particular sequence which is :-90,+90,+90,-90,-90,-90,-90,+90,+90,-90,-90,-90. All the above twelve angles form a cycle and thus starting of reading data from any other corner of I-section implies starting from corresponding angle in that cycle. Thus, irrespective of from where one starts, the angles measured in the way suggested will always fall in that particular sequence if it is an I-section.

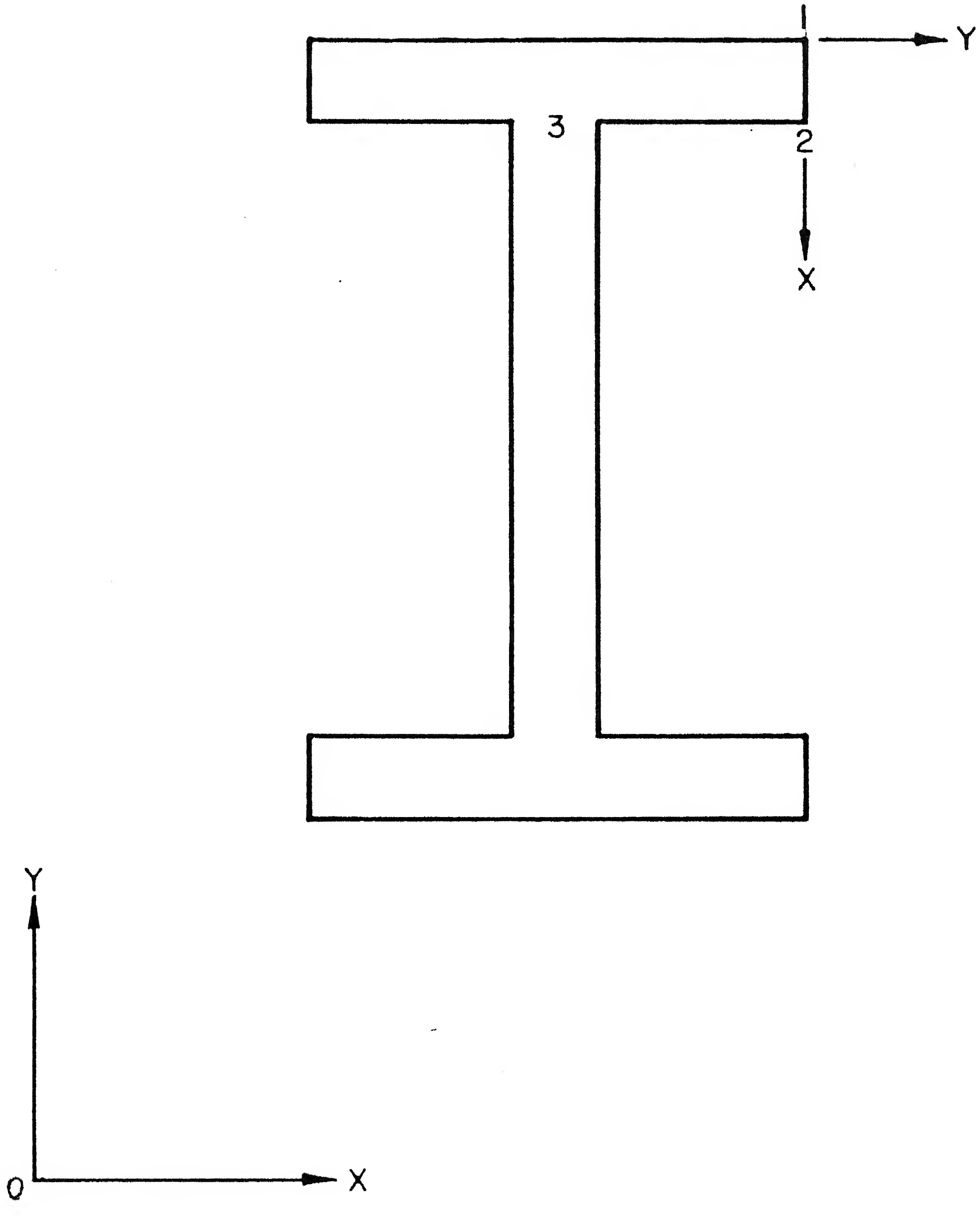


Figure 2.1: FIGURE OF AN I SECTION

Chapter 3

UPPER BOUND ANALYSIS OF HOT EXTRUSION

3.1 INTRODUCTION

In this chapter, an upper bound technique is presented for the extrusion of regular polygonal and rectangular sections through shaped dies. The work reported here is mainly concerned with the analysis of streamlined dies though the method is applicable to other curved dies such as parabolic, convex or concave die profiles. The method assumes a suitable form of stream function which satisfies the kinematical boundary conditions. The velocity field and strain rates are calculated from the assumed stream function.

3.2 PRINCIPLES OF UPPER BOUND TECHNIQUE

Among the various methods of analysis, the upper bound method is the most practical technique for simulating metal flow in simple deformation processes. This method can be used to estimate the deformation load and the forming pressure. Following steps are to be performed in this method:

a. Describe a family of admissible velocity fields which must satisfy the conditions of incompressibility, continuity and velocity boundaries.

b. Calculate the energy rates for deformation, internal shear and friction shear.

c. Calculate the total energy rate, and minimize it with respect to unknown parameters of velocity field formulation.

The load is then obtained by dividing the energy rate by the relative velocity between the die and the deforming material. The total energy rate J is given by

$$\begin{aligned} J &= load * dievelocity \\ J &= L\dot{V}_D = W_i + W_d + W_s \\ J &= \int_V \overline{\sigma}\dot{\overline{\epsilon}} dV + \int_S \tau \mid \Delta v \mid ds + \int_S \tau_i v_i ds \end{aligned} \tag{3.1}$$

where W_i, W_d, W_s are the energy rates for deformation, internal shear and friction respectively, L is the forming load, V is the volume of deforming material, v is the relative velocity between two zones of material when the velocity field has internal shear surfaces, S indicates the surface (internal or at die-material interface), v_i is the die material interface velocity in the i portion of the deforming material, $\tau=\bar{\sigma}/\sqrt{3}$ and $\tau_i=m_i\bar{\sigma}/\sqrt{3}$ =interface shear stress at the i portion of the deforming material.

It is found that the load calculated using the above equations is necessairly higher than the actual load and therefore represents an upper bound to actual forming load. Thus, the lower this upper bound load is, the better the prediction. Often the velocity field considered includes one or more parameters that are determined by minimizing the total energy rate with respect to these parameters. Thus, a somewhat better upper bound velocity field and solution are obtained. In general, with an increasing number of parameters in the velocity field the solution improves while the computations become more complex. Consequently, in the practical use of the upper bound method, compromises are made in selecting an admissible velocity field.

3.3 DESIGN CONSIDERATIONS FOR A DIE

In the extrusion process, the geometry of the die constitutes an important aspect of die design. The die profile determines the extent of redundant work done during the deformation. A profile which minimizes the redundant work, thereby minimizing the extrusion power, is called an optimal profile . Reddy[1] who compared eight different die shapes, came out with the conclusion that streamlined (third and fourth order polynomial) dies and the cosine die are the best . His results were based on the consideration of total extrusion power under optimal conditions. Streamlined dies have die surface constructed by smooth curved streamlines with zero gredient along the extruding direction at the entry and the exit of the die. This minimizes structural damage and work. Thus, streamlined dies have advantages such as reduced forming pressure and improved product quality.

3.4 DERIVATION OF A KINEMATICALLY ADMISSIBLE FLOW FIELD FOR A STREAM-LINED DIE FOR A POLYGONAL SECTION

As discussed in the last section, third- and fourth-order polynomial dies are the best ones but as fourth-order polynomial dies are comparatively difficult to manufacture and the corresponding gains are not significant. Therefore, only third-order polynomial dies which satisfy smooth entry and exit of the material has been considered in the persent analysis.

To construct the kinematically admissible velocity field for the extrusion of regular polygonal sections from cylinderical billets, the following assumptions have been made:

(a) The billet material passing through sector OEG (Fig 3.1) at the die entry goes through triangle LFH at the die exit, preserving the extrusion ratio.

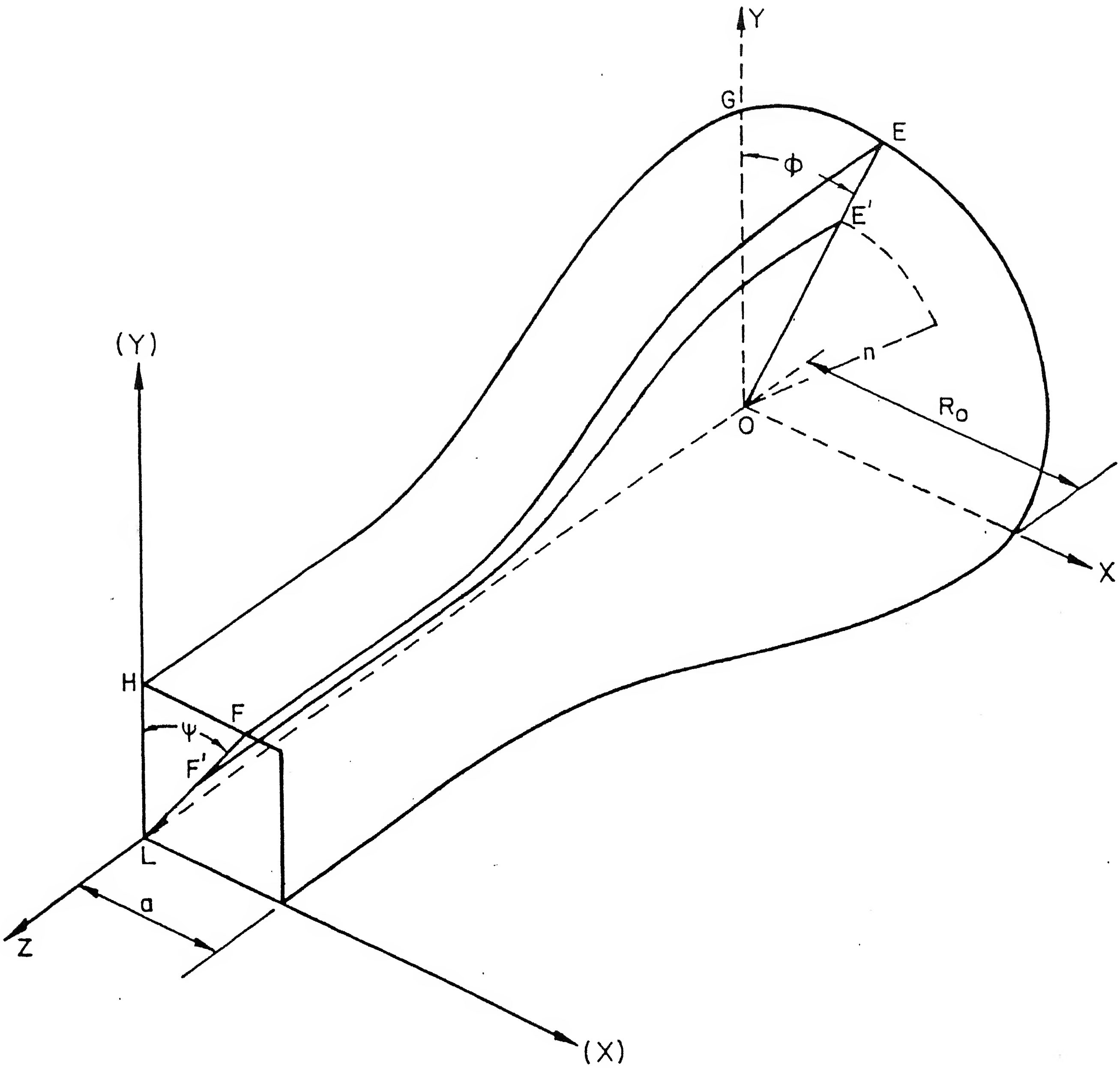


Figure 3.1: PROPOSED KINEMATICALLY ADMISSIBLE VELOCITY FIELD FOR REGULAR POLYGONAL SECTIONS

(b) Stream surface OEFL consists of a number of curved streamlines which start from a point E' at the entry and end at a corresponding point F' at the exit maintaining the proportionality of the position. The streamlines can be represented by cubic curves as follows:

$$\begin{aligned} x &= f_1(z) = a_1 z^3 + a_2 z^2 + a_3 z + a_4 \\ y &= f_2(z) = b_1 z^3 + b_2 z^2 + b_3 z + b_4 \\ z &= z \end{aligned} \tag{3.2}$$

The above set of equations show how x and y coordinates of streamlines vary with z coordinate. Here a_i and bi (i=1 to 4) are constants which are to be determined by the boundary conditions. As the streamlines do not produce any abrupt change of flow direction at the entrance and exit of die along the extrusion axis, the boundary conditions can be written as

$$x = n \sin\phi \;\; and \;\; \frac{\delta x}{\delta z} = 0 \;\; at \;\; z = 0 \tag{3.3}$$

$$y = n \cos\phi \;\; and \;\; \frac{\delta y}{\delta z} = 0 \;\; at \;\; z = 0 \tag{3.4}$$

$$x = \frac{n}{R_0} a \tan\psi \;\; and \;\; \frac{\delta x}{\delta z} = 0 \;\; at \;\; z = L \tag{3.5}$$

$$y = \frac{n}{R_0} a \;\; and \;\; \frac{\delta y}{\delta z} = 0 \;\; at \;\; z = L \tag{3.6}$$

There are in total eight constraints and eight boundary conditions therefore substituting these boundary conditions the values of these eight constants can be obtained. Hence,

$$\begin{aligned} x &= n \sin\phi + n\left(\frac{a}{R_0} c\phi - \sin\phi\right)\left[3(\frac{z}{L})^2 - 2(\frac{z}{L})^3\right] \\ y &= n \cos\phi + n\left(\frac{a}{R_0} - \cos\phi\right)\left[3(\frac{z}{L})^2 - 2(\frac{z}{L})^3\right] \\ z &= z \end{aligned} \tag{3.7}$$

Using continuity equation it can be shown that

$$\tan\psi = \frac{N}{\pi}\tan\left(\frac{\pi}{N}\right)\phi = c\phi \tag{3.8}$$

where

$$c = \frac{N}{\pi}\tan(\frac{\pi}{N})$$

The detailed derivation of eqn(3.8) is shown in Apendix A. The above set of derived equations(3.7) can also be written as

$$\begin{aligned}
x &= n\sin\phi + n\left(\frac{a}{R_0}c\phi - \sin\phi\right) f(z) \\
y &= n\cos\phi + n\left(\frac{a}{R_0} - \cos\phi\right) f(z) \\
z &= z
\end{aligned} \tag{3.9}$$

where

$$\begin{aligned}
f(z) &= 0 \;\; at \;\; z = 0 \\
f(z) &= 1 \;\; at \;\; z = L
\end{aligned} \tag{3.10}$$

Here, f(z) is a cubic function but can be any general function provided the function satisfies the boundary condition equations 3.10.

The set of equations (equations 3.7) describe not only the coordinates inside the plastically deforming region, but also the relationship between the cartesian and n,ϕ,z coordinate systems. To determine the velocity components for incompressible material, it is needed to calculate the determinant of the Jacobian of set of equations 3.7. Thus,

$$\begin{aligned}
J &= -n\left(\left[(1-f)^2 + c\left(\frac{a}{R_0}f\right)^2\right]\right. \\
&\quad + \left.\left[(1+c)(1-f)\frac{a}{R_0}f\right]\cos\phi + \left[c(1-f)\frac{a}{R_0}f\right]\phi\sin\phi\right)
\end{aligned} \tag{3.11}$$

$$J = -n * g(\phi, z) \tag{3.12}$$

where J is the Jacobian of the set of equations 3.7. The velocity components for incompressible material are determined as

$$v_x = \frac{n\left(\frac{a}{R_0}c\phi - sin\phi\right)f'(z)}{g(\phi,z)}v_0 \qquad (3.13)$$

$$v_y = \frac{n\left(\frac{a}{R_0} - cos\phi\right)f'(z)}{g(\phi,z)}v_0 \qquad (3.14)$$

$$v_z = \frac{1}{g(\phi,z)}v_0 \qquad (3.15)$$

Strain rate components are expressed as

$$\dot{\epsilon_{ij}} = \frac{1}{2}\left(\frac{\delta v_i}{\delta x_j} + \frac{\delta v_j}{\delta x_i}\right) \qquad (3.16)$$

Velocity field represented by equations (3.13) to (3.15) were analytically as well as numerically found to satisfy velocity boundary condition, i.e. $\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y} + \frac{\delta v_z}{\delta z} = 0$. The same velocity field was also found to satisfy the aforesaid incompressibility condition,that is, $\dot{\epsilon_{xx}} + \dot{\epsilon_{yy}} + \dot{\epsilon_{zz}} = 0$. Thus, the stringent requirements for constructing a kinematically admissible conditions are fulfilled for the above proposed velocity field.

3.5 UPPER BOUND SOLUTION

Streamlined dies do not produce velocity discontinuities at inlet and outlet of the dies, hence the term for power consumption due to velocity discontinuity will be absent. Thus, the total power consumption J^* required to deform cylindrical billets to regular polygonal sections through streamlined dies can be represented as the sum of the power consumed in plastic deformation(Wi) and in overcoming to the die surface friction (Ws). For the chosen section of analysis, J^* is obtained as

$$J^* = 2N(W_i + W_s) \qquad (3.17)$$

Here,

$$
\begin{aligned}
W_i &= \frac{2}{\sqrt{3}}\sigma_0 \int_V \left[\frac{1}{2}\dot{\epsilon_{ij}}\dot{\epsilon_{ij}}\right]^{\frac{1}{2}} dV \\
or, W_i &= \frac{2}{\sqrt{3}}\sigma_0 \int_0^L \int_0^{\phi_m} \int_0^{R_0} \left[\frac{1}{2}\dot{\epsilon_{ij}}\dot{\epsilon_{ij}}\right]^{\frac{1}{2}} \mid detJ \mid dnd\phi dz \qquad (3.18) \\
W_s &= m\frac{\sigma_0}{\sqrt{3}} \int_S \mid \Delta V \mid dS \\
or, W_s &= m\frac{\sigma_0}{\sqrt{3}} \int_0^L \int_0^{\phi_m} \left[V_x^2 + V_y^2 + V_z^2\right]^{\frac{1}{2}}_{n=R_0} \frac{1}{cos\alpha} \mid \frac{\delta(x,z)}{\delta(\phi,z)} \mid d\phi dz \qquad (3.19)
\end{aligned}
$$

Here α is the maximum angle of inclination of the die surface element with respect to the projected surface of the element onto x-z plane. L is the die length and R_0 is the billet radius. $\mid detJ \mid$ is the Jacobian of equation 3.7 and ϕ_m is determined by the symmetry of the die shape and has the maximum value of ϕ .

The total power consumption is obtained through the above volume and surface integration which are numerically carried out for a given value of yield stress of the material(σ_0) and a constant frictional factor (m). The power consumption can now be computed for an average pressure P_{av}, relative stress R_s, and relative die length R_L given by

$$P_{av} = \frac{J^*}{R_0^2 V_0 \pi} \qquad (3.20)$$

$$R_s = \frac{p_{av}}{\sigma_0} \qquad (3.21)$$

$$R_L = \frac{L}{R_0} \qquad (3.22)$$

3.6 OPTIMUM DIE LENGTH and OPTIMUM DIE PRESSURE

By plotting relative stress against relative die length for various sections and for different friction factors(fig. 5.1 to 5.5), it is observed that the optimum

die length always falls between $0.2R_0$ and $4R_0$. The, value of relative stress was checked for all values of die length between these two limits and the value of die length where relative stress was minimum was taken as the optimum die length. This minimum relative stress gives the value of pressure to be provided for carrying out the extrusion process.

3.7 KINEMATICALLY ADMISSIBLE FLOW FIELD FOR A STREAM-LINED DIE FOR A RECTANGULAR SECTION

In the case of a regular polygon ,it was possible to divide it into all symmetric parts and therefore only one such part was analysed. But in a rectangular cross-section, there are two sections which need different analysis. These sections are shown in fig.3.2. Sections denoted by + signs are identical and the same is true for sections with - signs. Thus sections OAB and OBC are analysed separately and it will be seen that all the proposed fields like velocity or strain rate fields are compatible at their common boundary.

Taking section LAH, the boundary conditions can be written as

$$x = nsin\phi \ and \ \frac{\delta x}{\delta z} = 0 \ ; \ z = 0 \tag{3.23}$$

$$y = ncos\phi \ and \ \frac{\delta y}{\delta z} = 0 \ ; \ z = 0 \tag{3.24}$$

and

$$x = \frac{n}{R_0} btan\psi_1 \ and \ \frac{\delta x}{\delta z} = 0 \ ; \ z = L \tag{3.25}$$

$$y = \frac{n}{R_0} b \ and \ \frac{\delta y}{\delta z} = 0 \ ; \ z = L \tag{3.26}$$

Also,

$$\tan\psi_1 = \left(\frac{4a}{\pi b}\right)\phi = c_1\phi \tag{3.27}$$

Refer to appendix for above derivation.

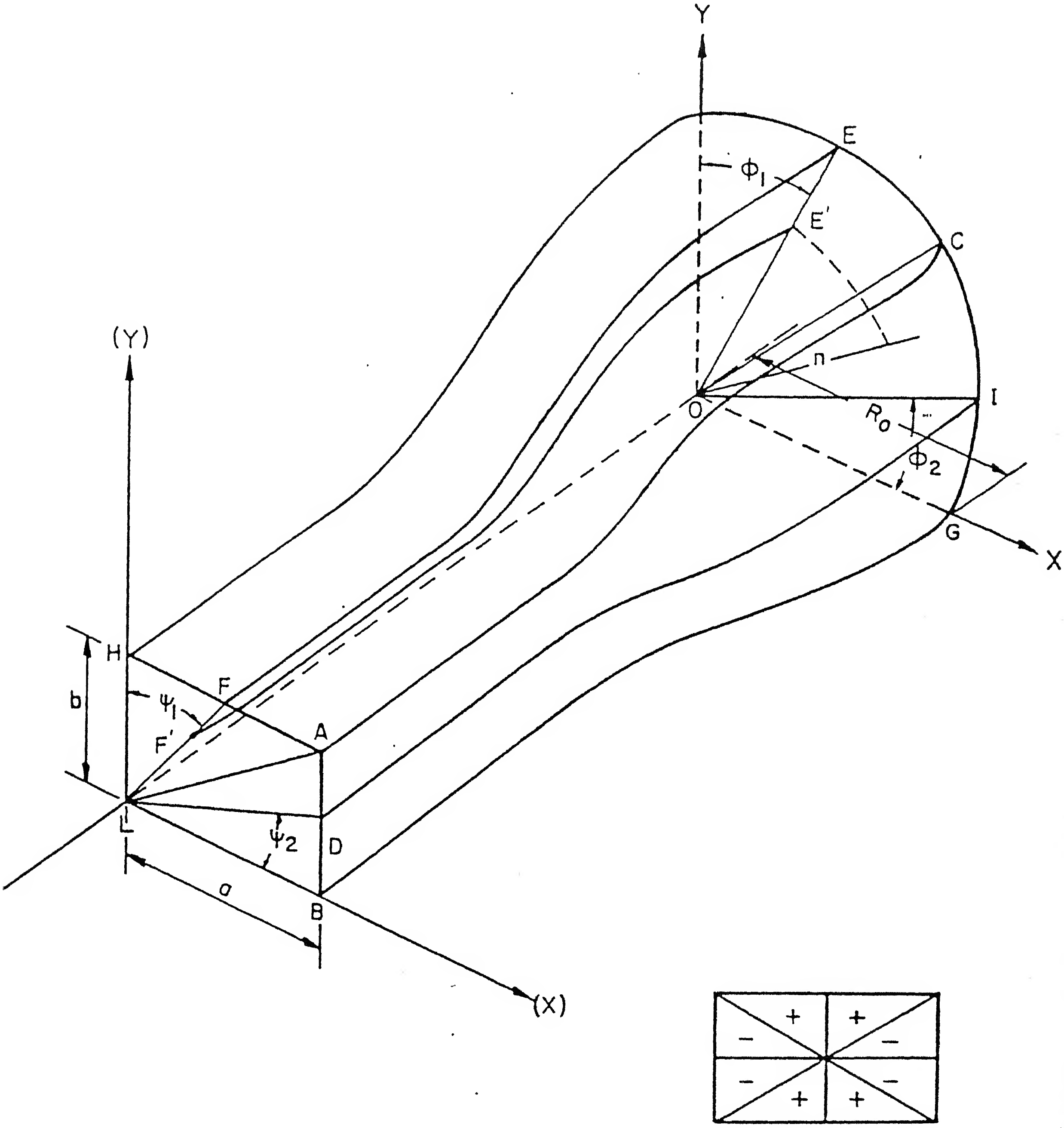


Figure 3.2: PROPOSED KINEMATICALLY ADMISSIBLE VELOCITY FIELD FOR A RECTANGULAR SECTION

Substituting all the above boundary conditions, one gets for the section LAH, cubic curves of the form

$$\begin{aligned} x &= n\sin\phi + n\left(\frac{b}{R_0}c_1\phi - \sin\phi\right)\left[3(\frac{z}{L})^2 - 2(\frac{z}{L})^3\right] \\ y &= n\cos\phi + n\left(\frac{b}{R_0} - \cos\phi\right)\left[3(\frac{z}{L})^2 - 2(\frac{z}{L})^3\right] \\ z &= z \end{aligned} \tag{3.28}$$

Simultaneously for the section LBA,we have the following boundary conditions

$$x = n\cos\phi \;\; and \;\; \frac{\delta x}{\delta z} = 0 \;;\;\; z = 0 \tag{3.29}$$

$$y = n\sin\phi \;\; and \;\; \frac{\delta y}{\delta z} = 0 \;;\;\; z = 0 \tag{3.30}$$

$$x = \frac{n}{R_0}a \;\; and \;\; \frac{\delta x}{\delta z} = 0 \;;\;\; z = L \tag{3.31}$$

$$y = \frac{n}{R_0}a\tan\psi_2 \;\; and \;\; \frac{\delta y}{\delta z} = 0 \;;\;\; z = L \tag{3.32}$$

In this case one gets the cubic curves of the form

$$\begin{aligned} x &= n\cos\phi + n\left(\frac{a}{R_0} - \cos\phi\right)\left[3(\frac{z}{L})^2 - 2(\frac{z}{L})^3\right] \\ y &= n\sin\phi + n\left(\frac{a}{R_0}c_2\phi - \sin\phi\right)\left[3(\frac{z}{L})^2 - 2(\frac{z}{L})^3\right] \\ z &= z \end{aligned} \tag{3.33}$$

Also,

$$\tan\psi_2 = \left(\frac{4b}{\pi a}\right)\phi = c_2\phi \tag{3.34}$$

From now onwards till the optimum die length is found, the whole procedure is almost same as was adopted for a regular polygonal section. Only few things needed to be adopted. We have $tan\psi = c\phi$, where c is a constant. Hence the maximum value of ϕ will correspond to the maximum value of ψ and maximum value of ψ will be $\tan^{-1}\left(\frac{a}{b}\right)$ in the first case and $tan^{-1}\left(\frac{b}{a}\right)$ in the second case.

Velocity and strain rates fields and total power consumption are obtained by numerical integrations in the same manner as for polygonal sections discribed earlier. The optimum die length and the corresponding extrusion pressure were also found obtained in a similar manner.

Chapter 4

IMPLEMENTATION

4.1 IMPLEMENTATION

The data exchange file of the given design is needed to be given as the input and the package will recognise the features of the design if it is one of those mentioned earlier. For this, certain geometrical constraints have been specified for each shape(in sections 2.1-2.4) and if the given shape satisfies any of the shape criteria then the corresponding shape is identified. If the design is of the rectangular or a regular polygonal section, then the package will proceed further, and it will provide optimum die length and the extrusion pressure. Otherwise it will stop here itself with just mentioning the shape of the input design. If it proceeds further then it also reads the basic geometrical data from the design such as length of each side of the shape. The upper bound analysis begins from here onwards. The assumed cubic curves representing the streamlines are subjected to boundary conditions and, therefore, all constants become known. From these curves we calculate the velocity components and the strain rates as discussed in the chapter 3. Whether continuity equation and the incompressibility conditions are satisfied or not, are also checked at this point. The total power consumption required to deform the cylindrical billets is obtained by calculating the power consumed due to plastic deformation and due to die surface friction as the former is the sum of these two. This involves

carrying out volume and surface integration with the help of Gauss-Legendre integration method. The total power consumption is converted into relative stress as discussed in the Chapter 3.

4.2 THE SOFTWARE PACKAGE

This work consists of recognizing the feature of the design first and then finding out the optimum die length and extrusion pressure for extruding the given shape. For attaining this, a software package in C language has been developed. The flow chart is shown in fig.4.1.

The data exchange file of the given shape is taken as input and the package recognizes the feature of the shape if it is one of those mentioned earlier. If the shape is of rectangular or regular polygonal section then the package proceeds further. At the same time, it would have read the basic geometrical data such as the length of the each side of the polygon. After this, the upper bound analysis starts and the user is required to feed the radius of the cylindrical billet which must be greater than a particular value. This critical value of radius is calculated on the basis of a geometrical data read from the product shape and is made known to the user. User is also asked for the value of friction factor. Finally the program gives the user the value of optimal die length and extrusion pressure needed for carrying out the extrusion process. The flow chart is explained below.

FLOW CHART EXPLANATION

1. It starts with the input of data exchange file and checks whether the file has line or circle statement.
2. If the file has line statements then the possibility of rectangular, regular or irregular polygonal and I-section is examined. And if the file has circle statements then option of circular or tubular section is looked for.
3. For the case where the file has line statements, firstly the possibility of rectangular section is checked. If the conditions for rectangular section as described in section 2.2 is satisfied, it is declared as rectangular section and

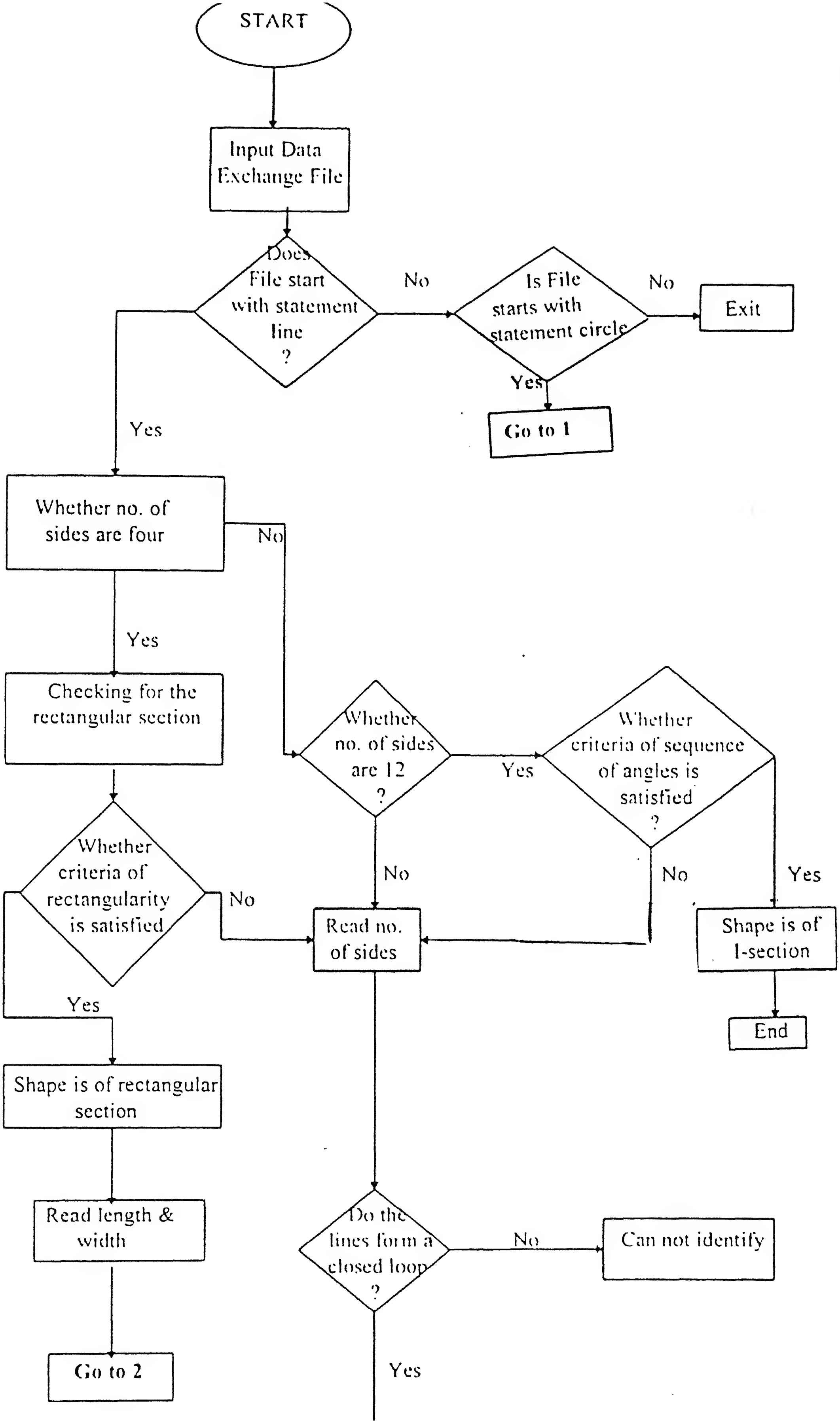
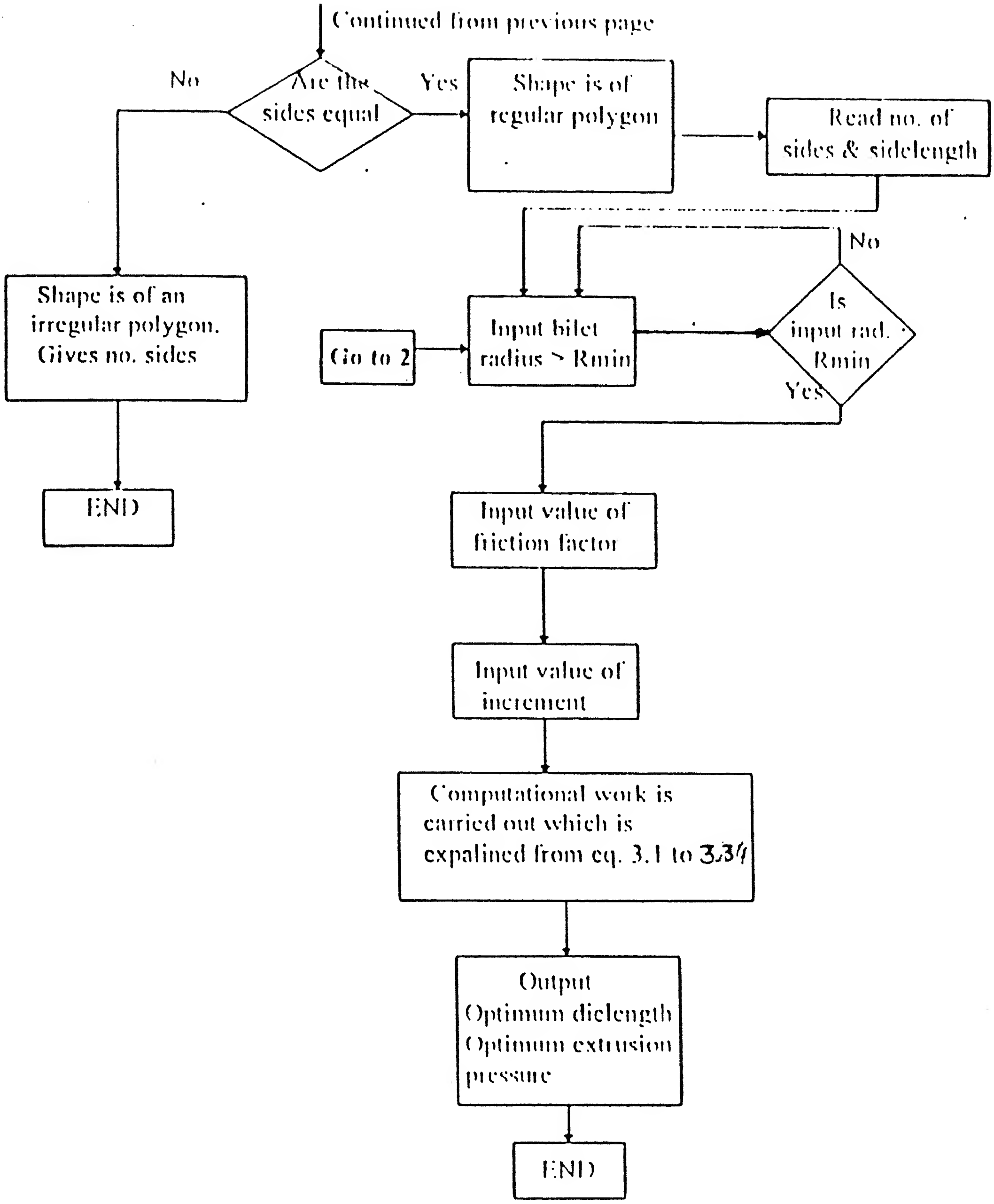
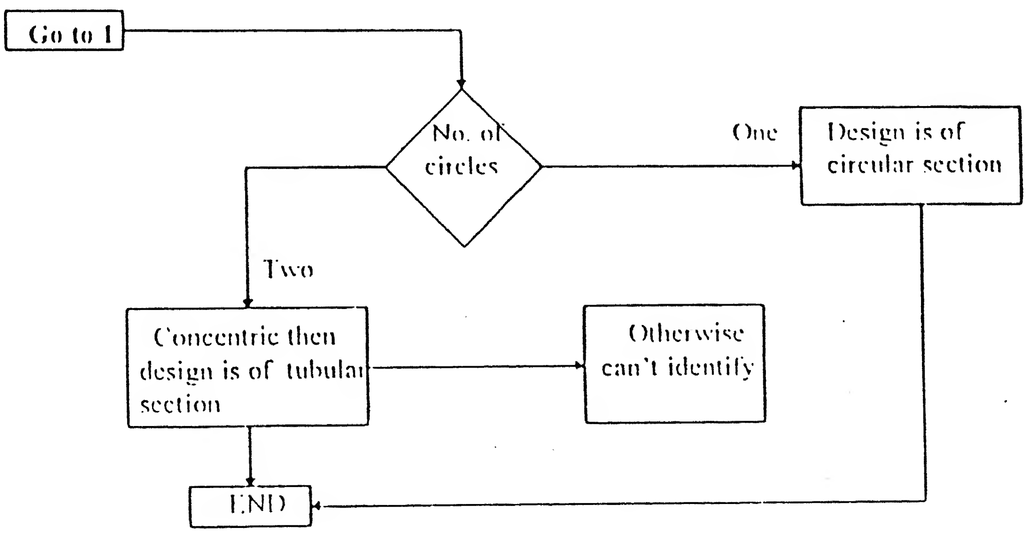


Figure 4.1: FLOW CHART OF THE SOFTWARE PROGRAMME



Continued from previous page
Are the sides equal
No
Yes
Shape is of regular polygon
Read no. of sides & sidelength
Shape is of an irregular polygon. Gives no. sides
END
Go to 2
Input bilet radius > Rmin
Is input rad. Rmin
No
Yes
Input value of friction factor
Input value of increment
Computational work is carried out which is expalined from eq. 3.1 to 3.34
Output Optimum dielength Optimum extrusion pressure
END



Go to 1
No. of circles
One
Design is of circular section
Two
Concentric then design is of tubular section
Otherwise can't identify
END

its dimension such as length and width are read. Other inputs such as value of friction factor and value of increment is asked for. All the computations are then done and the output is given which consists of optimum die length and minimum extrusion pressure.
4. If criteria of rectangular section is not satisfied then possibility of I-section is examined. If all the constraints as described in section 2.4 are satisfied then the shape is that of I-section. In this case the program ends here.
5. When the possibilities of I-section and rectangular are exhausted, then the conditions for regular and irregular polygonal sections are tested. If the shape turns out to be irregular polygon then programme stops here itself with just mentioning the shape and the number of sides. If the sides are equal then the shape is regular polygon and the further steps are same as in step3.
6. If the file has circle statements and dimension of only one circle is given then it is a circular section. And if there are two circles with their centres coincident and radii different then it is tubular section. The program ends here with just mentioning the shape of the section.

4.3 EXAMPLES

4.3.1 I-SECTION

An I-section of the size given in the input file is shown in Fig.4.2. Steps are as follows:

1. Input file is given.
2. The coordinates of initial and end points of all the lines are given in the starting of the output file.
3. Programme then calculates the lengths of the sides.
4. It checks for all the shapes and as the criteria for I-section is being satisfied, it identifies the shape as that of I-section.

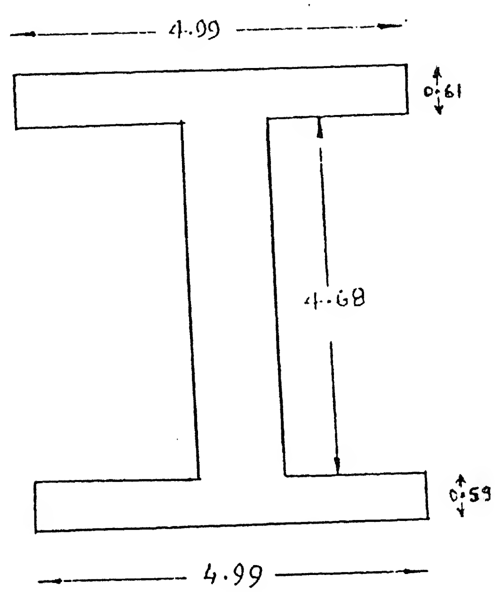


Figure 4.2: An I-section of the size given in the input file

```
##################################################
      INPUT FILE 'di'
##################################################
LINE
  8
0
 10
4.34948
 20
8.421462
 30
0.0
 11
9.342373
 21
8.421462
 31
0.0
  0
LINE
  8
0
 10
9.342373
 20
8.421462
 30
0.0
 11
9.342373
 21
7.803373
 31
0.0
  0
LINE
  8
0
 10
9.342373
 20
7.803373
 30
0.0
 11
7.56655
 21
7.803373
 31
0.0
  0
LINE
  8
0
 10
7.56655
 20
7.803373
 30
0.0
 11
7.56655
 21
3.116199
 31
0.0
  0
LINE
  8
 20
3.116199
 30
0.0
 11
9.342373
 21
3.116199
 31
0.0
  0
LINE
  8
0
 10
9.342373
 20
3.116199
 30
0.0
 11
9.342373
 21
2.523863
 31
0.0
  0
LINE
  8
0
 10
9.342373
 20
2.523863
 30
0.0
 11
4.34948
 21
2.523863
 31
0.0
  0
LINE
  8
0
 10
4.34948
 20
2.523863
 30
0.0
 11
4.34948
 21
3.116199
 31
0.0
  0
LINE
  8
0
 10
4.34948
 20
0.0
 11
6.485615
 21
3.116199
 31
0.0
  0
LINE
  8
0
 10
6.485615
 20
3.116199
 30
0.0
 11
6.485615
 21
7.803373
 31
0.0
  0
LINE
  8
0
 10
6.485615
 20
7.803373
 30
0.0
 11
4.34948
 21
7.803373
 31
0.0
  0
LINE
  8
0
 10
4.34948
 20
7.803373
 30
0.0
 11
4.34948
 21
8.421462
 31
0.0
  0
```

```
******************************************
ing thes
******************************************
--------------------------------------------------------------
: filename
--------------------------------------------------------------

ı are the coordinates of the vertices in the following order x1 x2

)480 9.342373 8.421462 8.421462

2373 9.342373 8.421462 7.803373

2373 7.566550 7.803373 7.803373

5550 7.566550 7.803373 3.116199

5550 9.342373 3.116199 3.116199

2373 9.342373 3.116199 2.523863

2373 4.349480 2.523863 2.523863

9480 4.349480 2.523863 3.116199

9480 6.485615 3.116199 3.116199

5615 6.485615 3.116199 7.803373

5615 4.349480 7.803373 7.803373

9480 4.349480 7.803373 8.421462
owing are the length of 1 th side
2893
owing are the length of 2 th side
8089
owing are the length of 3 th side
5823
owing are the length of 4 th side
7174
owing are the length of 5 th side
'5823
owing are the length of 6 th side
)2336
owing are the length of 7 th side
)2893
owing are the length of 8 th side
)2336
owing are the length of 9 th side
36135
lowing are the length of 10 th side
37174
lowing are the length of 11 th side
36135
lowing are the length of 12 th side
18089

--------------------------------------------------------------
en shape has a I cross-section

--------------------------------------------------------------
```

```
########################################################
        INPUT FILE 'dr'
########################################################
LINE
  8
0
 10
3.139861
 20
8.395709
 30
0.0
 11
11.735874
 21
8.395709
 31
0.0
  0
LINE
  8
0
 10
11.735874
 20
8.395709
 30
0.0
 11
11.735874
 21
5.562801
 31
0.0
  0
LINE
  8
0
 10
11.735874
 20
5.562801
 30
0.0
 11
3.139861
 21
5.562801
 31
0.0
  0
LINE
  8
0
 10
3.139861
 20
5.562801
 30
0.0
 11
3.139861
 21
8.395709
 31
0.0
```

```
[$PWD] $  run
******************************************************
Running thes
******************************************************
---------------------------------------------------------------------------
input filename
---------------------------------------------------------------------------
dr
given are the coordinates of the vertices in the following order x1 x2 y1 y2
1
3.139861 11.735874 8.395709 8.395709
2
11.735874 11.735874 8.395709 5.562801
3
11.735874 3.139861 5.562801 5.562801
4
3.139861 3.139861 5.562801 8.395709
length=8.596013
breadth=2.832908
---------------------------------------------------------------------------


given shape has a rectangular cross-section

---------------------------------------------------------------------------
******************************************************
Running r4
******************************************************
give the  length of the rectangular section
give the  breadth of the rectangular section
8.596013
2.832908
-------------------------------------------------------------------------------
give the  value of the friction factor
-------------------------------------------------------------------------------
1
-------------------------------------------------------------------------------
Enter the radius of Cylindrical billet
Radius of Billet should be greater than 4.525395
-------------------------------------------------------------------------------
5
GIVEN R0 = 5.000000
-------------------------------------------------------------------------------
Give the value of increment over which you want the variation of
different values
-------------------------------------------------------------------------------
0.5
number of iterations==18
value of rel stress in 1 iteration
4.575881  at dielength 1.000000

value of rel stress in 2 iteration
3.857671  at dielength 1.500000

value of rel stress in 3 iteration
3.573705  at dielength 2.000000

value of rel stress in 4 iteration
3.460806  at dielength 2.500000

value of rel stress in 5 iteration
3.432642  at dielength 3.000000

value of rel stress in 6 iteration
3.452661  at dielength 3.500000

value of rel stress in 7 iteration
3.502732  at dielength 4.000000value of rel stress in 8 iteration
3.572817  at dielength 4.500000
```

```
value of rel stress in 9 iteration
3.656903  at dielength 5.000000

value of rel stress in 10 iteration
3.751160  at dielength 5.500000

value of rel stress in 11 iteration
3.853034  at dielength 6.000000

value of rel stress in 12 iteration
3.960755  at dielength 6.500000
value of rel stress in 13 iteration
4.073056  at dielength 7.000000

value of rel stress in 14 iteration
4.189012  at dielength 7.500000

value of rel stress in 15 iteration
4.307927  at dielength 8.000000

value of rel stress in 16 iteration
4.429271  at dielength 8.500000

value of rel stress in 17 iteration
4.552632  at dielength 9.000000

value of rel stress in 18 iteration
4.677687  at dielength 9.500000

------------------------------------------------------------------
optimum die-length = 3.000000
Relstress=  3.432642
------------------------------------------------------------------
```

```
[$PWD] $  run
*************************************************************
Running thes
*************************************************************
--------------------------------------------------------------------------------
input filename
--------------------------------------------------------------------------------
dhx
given are the coordinates of the vertices in the following order x1
1
7.566550 11.324089 8.910783 7.751866
2
11.324089 11.324089 7.751866 4.326623
3
11.324089 7.566550 4.326623 2.652632
4
7.566550 3.989168 2.652632 4.326623
5
3.989168 3.989168 4.326623 7.751866
6
3.989168 7.566550 7.751866 8.910783
Following are the length of 1 th side
3.932199
Following are the length of 2 th side
3.425243
Following are the length of 3 th side
4.113556
Following are the length of 4 th side
3.949672
Following are the length of 5 th side
3.425243
Following are the length of 6 th side
3.760419
--------------------------------------------------------------------------------
given x-section is hexa
--------------------------------------------------------------------------------
```

INPUT FILE 'hex1'

```
LINE
  8
0
 10
5.0
 20
5.0
 30
0.0
 11
6.0
 21
5.0
 31
0.0
  0
LINE
  8
0
 10
6.0
 20
5.0
 30
0.0
 11
6.5
 21
5.866025
 31
0.0
  0
LINE
  8
0
 10
6.5
 20
5.866025
 30
0.0
 11
6.0
 21
6.732051
 31
0.0
  0
LINE
  8
0
 10
6.0
 20
6.732051
 30
0.0
 11
5.0
 21
6.732051
 31
0.0
  0
LINE
  8
0
 30
0.0
 11
4.5
 21
5.866025
 31
0.0
  0
LINE
  8
0
 10
4.5
 20
5.866025
 30
0.0
 11
5.0
 21
5.0
 31
0.0
  0
```

```
[$PWD] $  run
*****************************************************
Running thes
*****************************************************
-----------------------------------------------------------------------
input filename
-----------------------------------------------------------------------
hex1
given are the coordinates of the vertices in the following order x1 x2
1
5.000000 6.000000 5.000000 5.000000
2
6.000000 6.500000 5.000000 5.866025
3
6.500000 6.000000 5.866025 6.732051
4
6.000000 5.000000 6.732051 6.732051
5
5.000000 4.500000 6.732051 5.866025
6
4.500000 5.000000 5.866025 5.000000
Following are the length of 1 th side
1.000000
Following are the length of 2 th side
1.000000
Following are the length of 3 th side
1.000000
Following are the length of 4 th side
1.000000
Following are the length of 5 th side
1.000000
Following are the length of 6 th side
1.000000
-----------------------------------------------------------------------
given x-section is hexagonal having all sides equal
-----------------------------------------------------------------------


length of each side= 1.000000
*****************************************************
Running n4
*****************************************************
please give the number of sides
give the length of the final shape,i.e regular polygon

1.000000
-----------------------------------------------------------------------
give the value of friction factor
-----------------------------------------------------------------------

0.6
-----------------------------------------------------------------------
give the radius of the cylinderical billet and that should be greater
than 1.000027
-----------------------------------------------------------------------
0.5
-----------------------------------------------------------------------
Radius of cylinderical billet should be greater than 1.000027
Reenter the radius of billet
-----------------------------------------------------------------------
2.0
-----------------------------------------------------------------------
Give the value of increment over which you want the variation
of different values
-----------------------------------------------------------------------
0.5
Number of iterations==7
 value of rel stress in 1 iteration
7.371751  at die length 0.400000
```

```
lue of rel stress in 4 iteration
36556  at die length 1.900000

lue of rel stress in 5 iteration
35867  at die length 2.400000

lue of rel stress in 6 iteration
01715  at die length 2.900000

lue of rel stress in 7 iteration
03245  at die length 3.400000

------------------------------------------------------------------
:imum die-length= 2.400000
.Stress=  4.035867
------------------------------------------------------------------
```

```
##########################################################
        INPUT FILE 'dt'
##########################################################

CIRCLE
  8
0
 10
6.794453
 20
6.361166
 30
0.0
 40
3.173897
  0
CIRCLE
  8
0
 10
6.794453
 20
6.361166
 30
0.0
 40
2.349364
  0
[$PWD] $  run
*****************************************************
Running thes
*****************************************************
-------------------------------------------------------------------
input filename
-------------------------------------------------------------------
dt
Following are the x-coor, y-coor and radius of the circle
6.794453 6.361166 3.173897
Following are the x-coor, y-coor and radius of the circle
6.794453 6.361166 2.349364
-------------------------------------------------------------------
given shape has a tubular cross-section

-------------------------------------------------------------------
```

4.3.2 RECTANGULAR-SECTION

An rectangular section of the size given in the input file is shown in fig.4.3.

1.The input file is given.
2. The coordinates of initial and end points of all the lines are given in the starting of the output file.
3. Programme then calculates the lengths of the sides.
4. It checks for all the shapes and as the criteria for rectangular section is being satisfied, it identifies the shape as that of rectangular section.
5. It proceeds further and reads the length and width of the rectangular section.
6. It asks for the values of friction factor,radius of cylinderical billet and the value of increment.
7. Number of iterations that are to be carried out are calculated and values of relative stress over different dielengths are calculated.
8. Optimum dielength corresponding to minimum relative stress is given as the output. In this case, for length=8.59, breadth=2.83, billetradius=5, friction factor=1 and value of increment=0.5, the optimum die length is 3 and relative stress is 3.43.

4.3.3 IRREGULAR POLYGONAL SECTION

An irregular polygonal section of the size given in the input file is shown in fig.4.4.

1.The input file is given.
2. The coordinates of initial and end points of all the lines are given in the starting of the output file.
3. Programme then calculates the lengths of the sides.
4. It checks for all the shapes and as the criteria for irregular hexagonal section is being satisfied, it identifies the shape as that of irregular hexagonal section.

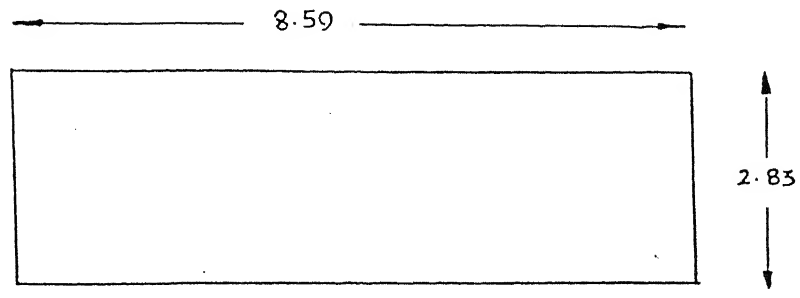


Figure 4.3: Rectangular section of the size given in the input file

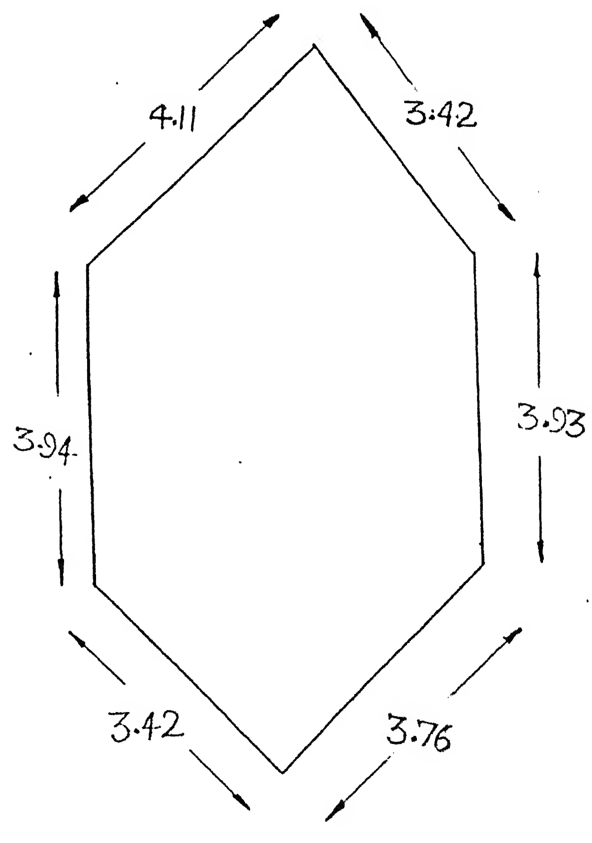


Figure 4.4: Irregular polygonal section of the size given in the input file

4.3.4 REGULAR POLYGONAL SECTION

An regular polygonal section of the size given in the input file is shown in fig.4.5.

1.The input file is given.
2. The coordinates of initial and end points of all the lines are given in the starting of the output file.
3. Programme then calculates the lengths of the sides.
4. It checks for all the shapes and as the criteria for regular hexagonal section is being satisfied, it identifies the shape as that of regular hexagonal section.
5. It proceeds further and reads the length of each side of the regular hexagonal section.
6. It asks for the values of friction factor,radius of cylinderical billet and the value of increment.
7. Number of iterations that are to be carried out are calculated and values of relative stress over different dielengths are calculated.
8. Optimum dielength corresponding to minimum relative stress is given as the output. In this case, for length of each side=1, billetradius=2, friction factor=0.6 and value of increment=0.5, the optimum die length is 2.4 and relative stress is 4.03.

4.3.5 TUBULAR SECTION

An tubular section of the size given in the input file is shown in fig.4.6.

1.The input file is given.
2. The coordinates of centres of the circles are given in the starting of the output file.
3. It checks for all the shapes and as the criteria for tubular section is being satisfied, it identifies the shape as that of tubular section.

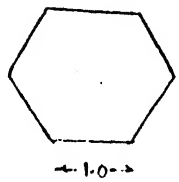


Figure 4.5: Regular polygonal section of the size given in the input file

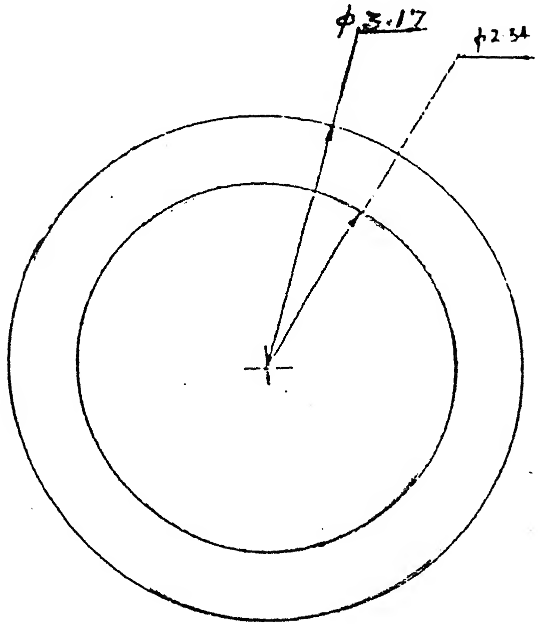


Figure 4.6: Tubular section of the size given in the input file

Chapter 5

RESULTS AND DISCUSSIONS

Apart from developing a CAPP system for the extrusion process, some intersetting observations are also made from the upper bound analysis of the polygonal and rectangular section. In Fig. 5.1, a plot of total Power (J), Power consumed due to plastic deformation (W_i) and that due to friction (W_s) against relative die length for a hexagonal section is shown. All of the power components have been converted into net stress by dividing them by $\pi R_o^2 V_o \sigma_o$ as already explained. Fig. 5.2 shows the same variations for the rectangular section. These figures explain the contribution of each power component to the total extrusion power. It is readily seen that power consumed due to plastic deformation is dominant in particular for the shorter length of the die. This power component decreases drastically with the increase of the die length. This change is attributed to the change of the flow direction inside the plastically deforming zone.

Fig. 5.3 shows the effect of shape of polygon on the extrusion stress with the die length. There is a decrease in extrusion stress with the increase in number of sides. It is because of decrease in power consumption due to both plastic deformation and friction. In the former case, as the number of sides increase, the change in shape is from drastic to gradual and hence the decrease in power consumption with the number of sides. Similarly in the latter case there is a decrease in surface area with the number of sides for a given area reduction

and therefore a corresponding fall in power consumption.

In Fig. 5.4, the effect of friction factor on the total power consumed with the relative die length for a square section is shown. The optimal die length which produces the minimal extrusion stress decreases with the increase of friction. It happens as the increase of friction will lead to increased share of the power component that due to friction in the total power consumption. In this condition, smaller die length will have comparatively less power consumption due to friction but slightly enhanced power consumption due to plastic deformation. Therefore, optimal die length will be that one where decrease in former balances increase in the latter. The same result is obtained for rectangular section as shown in Fig.5.5.

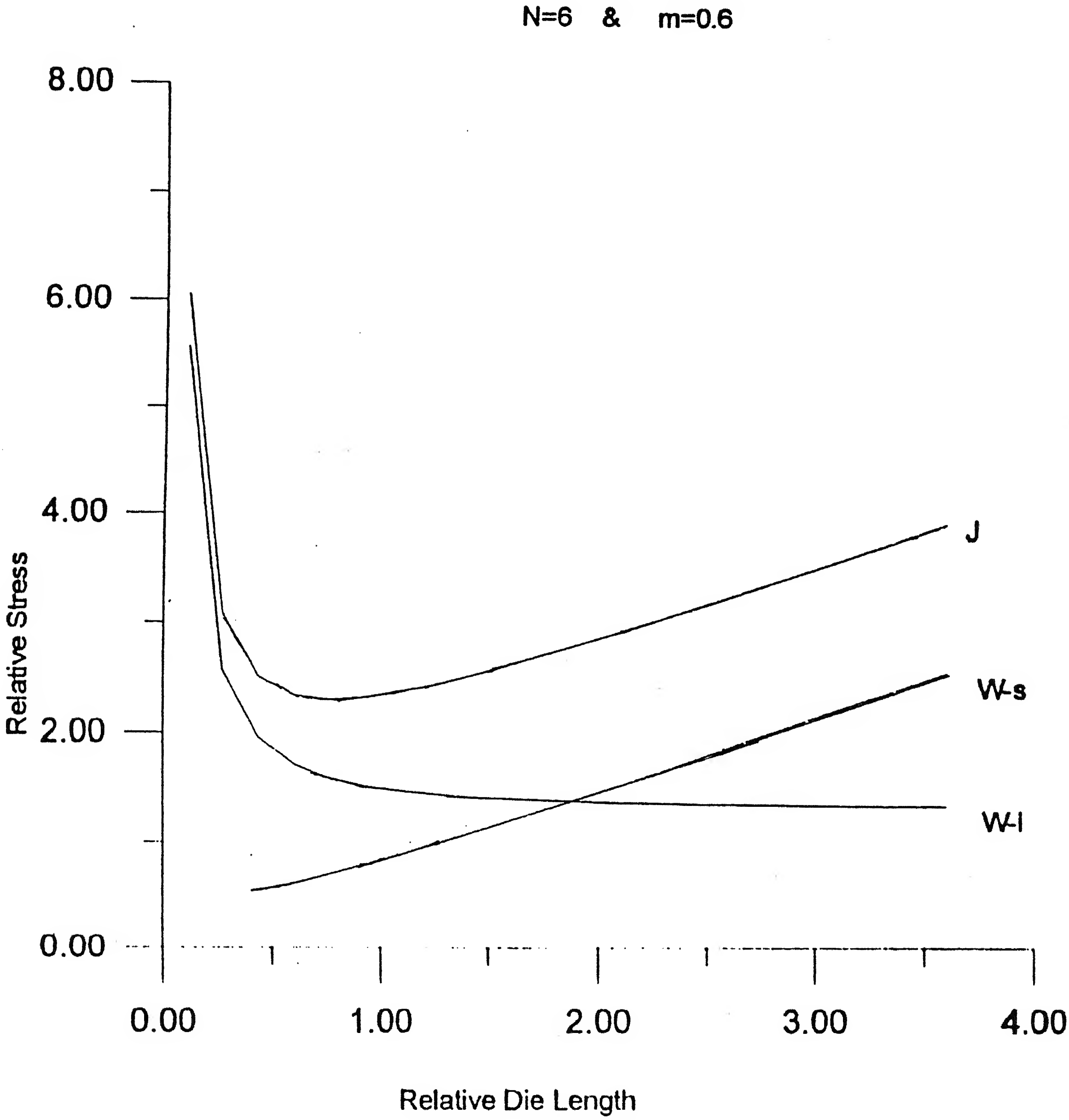


Figure 5.1: Variation of J,W_i and W_s against relative die length for a hexagonal section

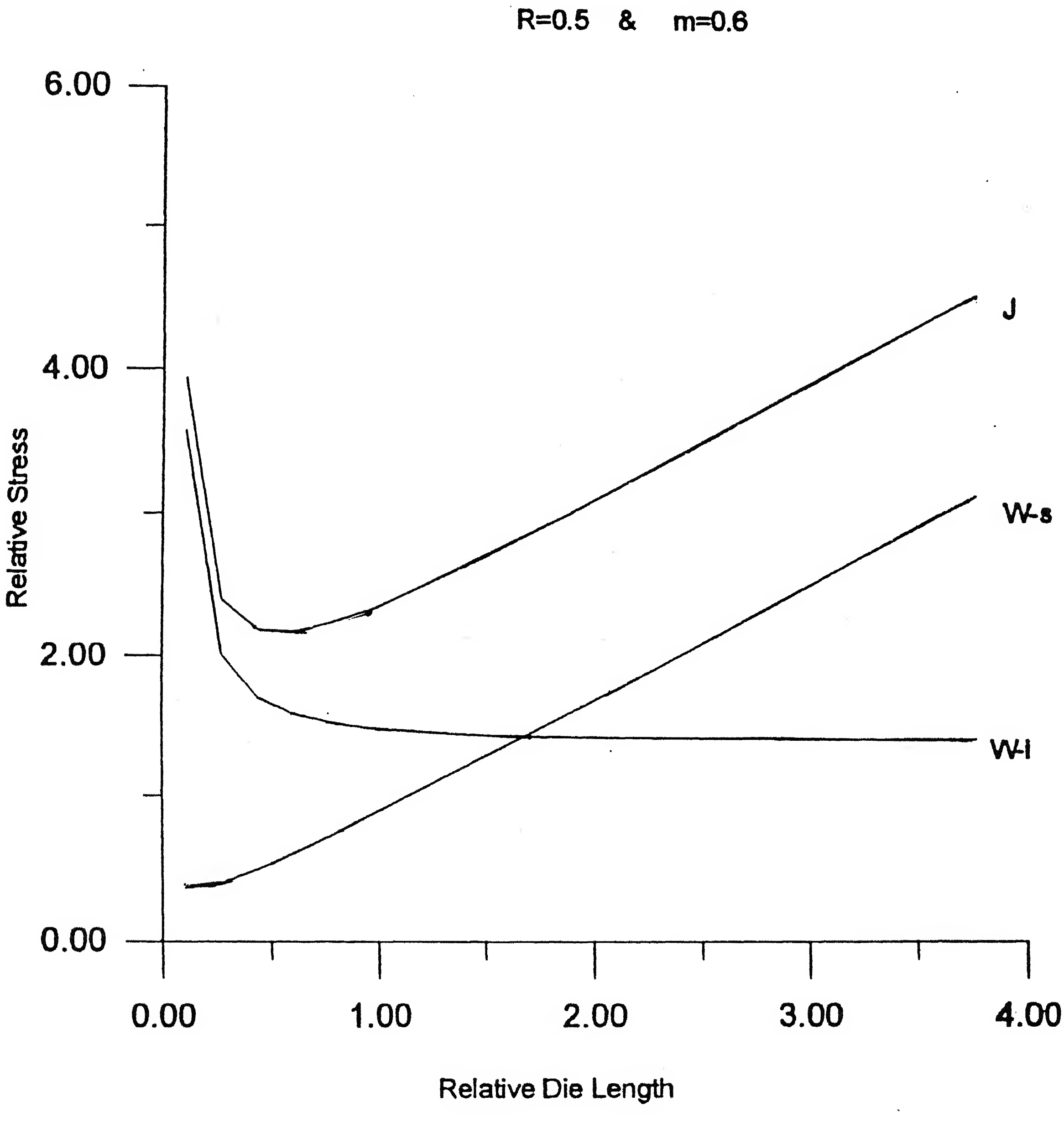


Figure 5.2: Variation of J,W_i and W_s against relative die length for a rectangular section

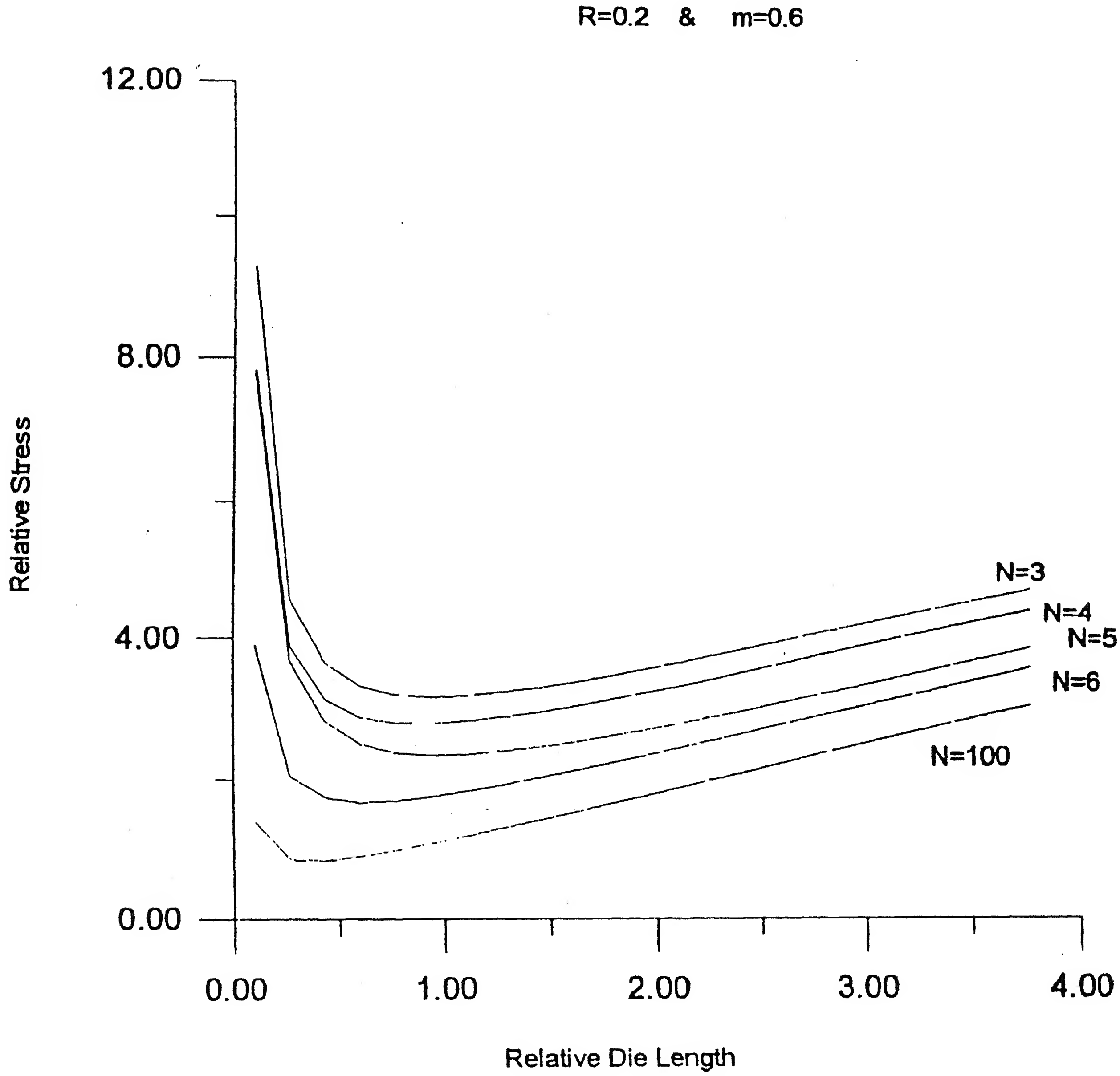


Figure 5.3: Effect of N on total power consumption with the relative die length for a regular polygonal section

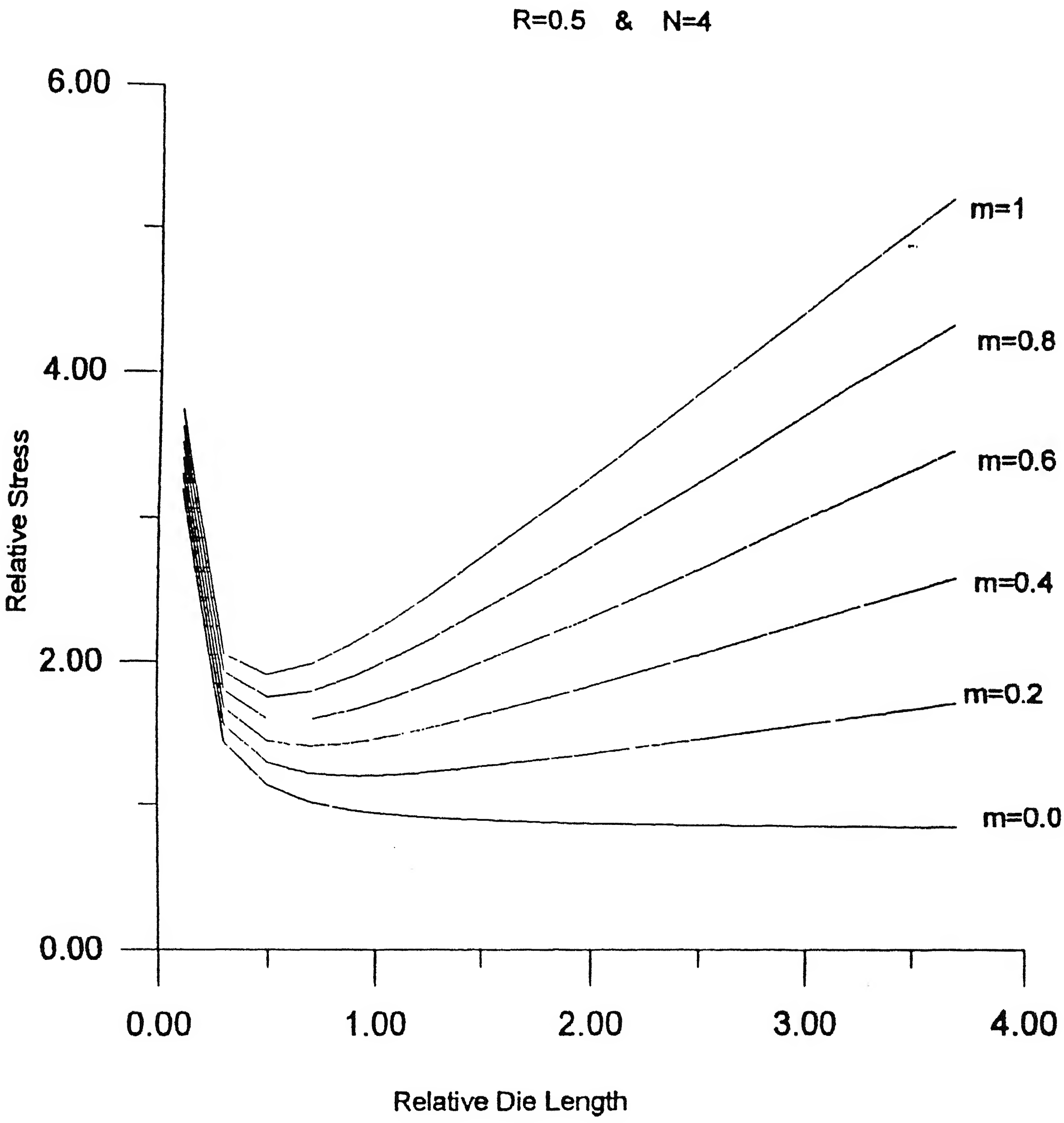


Figure 5.4: EFFECT OF FRICTION FACTOR ON TOTAL POWER CONSUMPTION WITH THE RELATIVE DIE LENGTH FOR A SQUARE SECTION

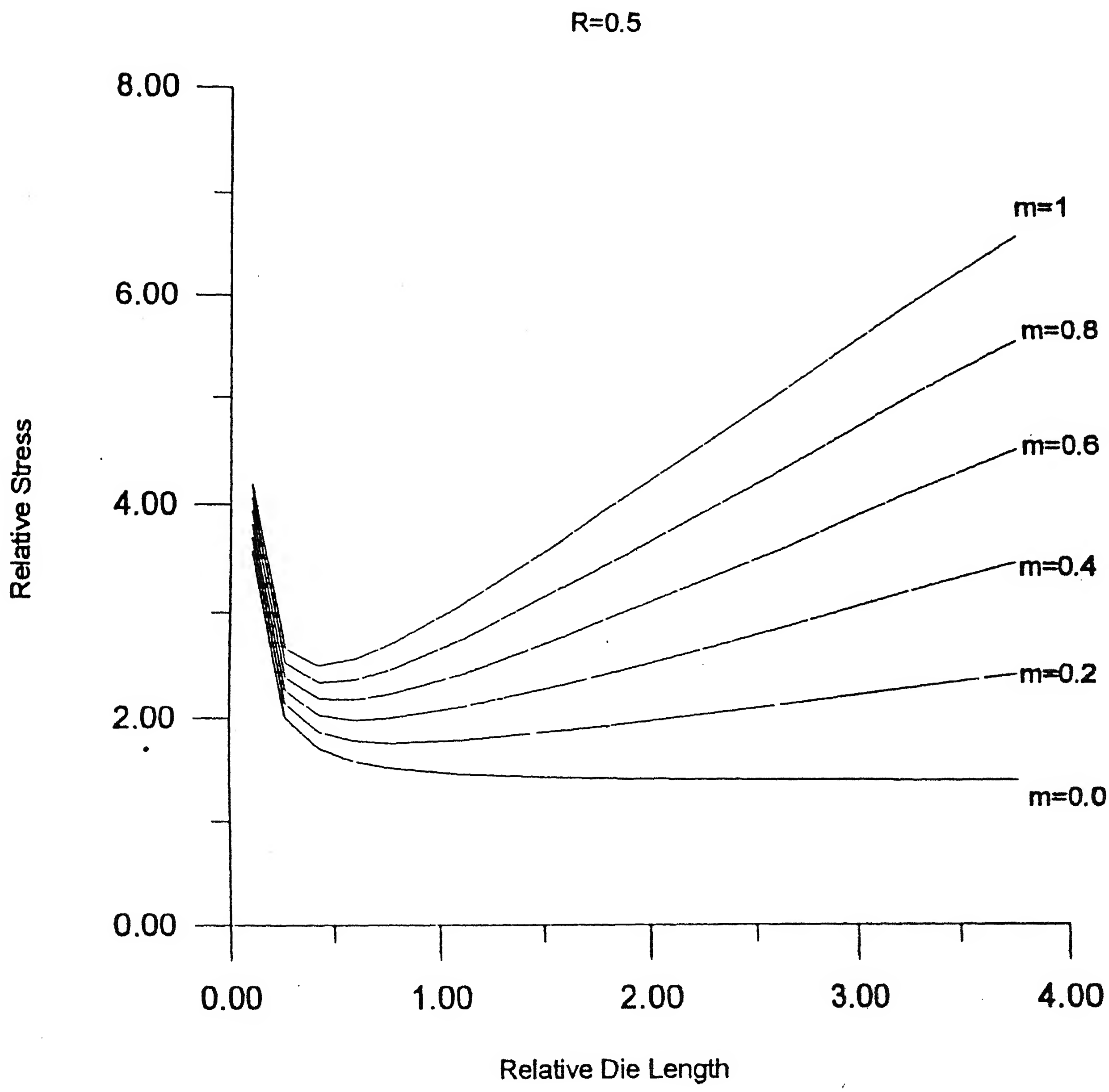


Figure 5.5: EFFECT OF FRICTION FACTOR ON TOTAL POWER CONSUMPTION WITH THE RELATIVE DIE LENGTH FOR A RECTANGULAR SECTION

5.1 SCOPE FOR FUTURE WORK

1. In this work, feature recognition process is provided for only few shapes. Feature recognition process for other shapes can be developed based on the methods proposed to make this CAPP system more extensive.

2. Similarly upper bound solution is provided for regular polygonal and rectangular section. The same analysis can be done for other shapes also.

3. Instead of streamlined die profile, cosine die profile can also be used for the analysis work.

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1.

$$\tan\psi = \frac{N}{\pi}\tan\left(\frac{\pi}{N}\right)\phi \tag{.1}$$

Two assumption have been made here:

(1) The material of the billet passing through sector OEG at the die entry goes through tringle LFH at the die exit,preserving the extrusion(or the drawing ratio).

(2) Stream surface OEFL consists of a number of curved streamline, which start from a point(sayE')at the entry and end at corresponding point(F')at the exit maintaining the proportionality of the position.

According to assumptions

$$\begin{aligned} \frac{Area of sector OEG}{Area of billet} &= \frac{Area of sector LFH}{Area of product cross-section} \qquad (.2)\\ or,\ \frac{\frac{1}{2}R^2\phi}{\pi R^2} &= \frac{\frac{1}{2}(\frac{a}{2}\cot\frac{\pi}{N})^2\tan\psi}{2N(\frac{1}{2}(\frac{a}{2})^2\cot\frac{\pi}{N})}\\ or,\ \frac{\phi}{2\pi} &= \frac{\tan\psi}{2N\tan\frac{\pi}{N}}\\ or,\ \tan\psi &= \frac{N}{\pi}\tan\left(\frac{\pi}{N}\right)\phi \end{aligned}$$

2.

$$\tan\psi_1 = \frac{4a}{\pi b}\phi \tag{.3}$$

The assumpation made above also apply here and refer to the Fig 3.2.

$$\begin{aligned} \frac{\frac{1}{2}b^2\tan\psi_1}{4ab} &= \frac{\frac{1}{2}R^2\phi}{\pi R^2} \qquad (.4)\\ or,\ \tan\psi_1 &= \frac{4a}{\pi b}\phi \end{aligned}$$

(.5)